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Soft Lattice Truss Static Polynomial Response Using Energy Methods

Winfred S. Kenner*
NASA Langley Research Center, Hampton, Virginia 23681-0001
and
Norman F. Knight Jr.†
Old Dominion University, Norfolk, Virginia 23529-0247

A deterministic methodology is presented for developing closed-form deflection equations for two-dimensional and three-dimensional lattice structures. Two types of lattice structures are studied: beams and soft lattices. Castigliano's second theorem, which entails the total strain energy of a structure, is utilized to generate highly accurate results. Derived deflection equations provide new insight into the bending and shear behavior of the two types of lattices, in contrast to classic solutions of similiar lattice truss structures.

Introduction

TRUSSES are rigid skeletal frameworks utilized to provide support for structures or equipment. They are generally composed of long slender members. Typically these members are joined with pin-connectors. Some attractive structural features of trusses are their low material-to-load-carrying characteristics relative to solid beams, ease of construction, and predictable behavior while incurring load. Truss designers rely on geometry, redundancy, and/or arch action to tailor and to optimize trusses for various load applications. These and other design parameters play a crucial role in the performance of cranes, bridges, domes, and space-based structures. Various truss applications require designer evaluation of behavior for operational loading, vibrational excitations, or loads during truss construction. For preliminary analysis or conceptual studies, designers often study single- or double-layer planar lattice truss structures, which have regular and patterned geometries, to gain insight into their structural behavior. Lattice structures are attractive for stiffness and vibration analysis methods because their repetitive geometries are represented by mathematical models and numerical programs in a more accurate manner than trusses with curvature or local variations. However, even the study of lattices encompasses a wide range of analysis techniques. Many reports and books have been published on lattice stiffness analysis, tailoring for load-transfer efficiency, and vibration response prediction.[1–8] One of the most common types of analysis for lattice structures is the continuum approach in which a lattice's stiffness properties are represented by an equivalent continuum model. However, continuum analysis is most useful for large lattices with many repeating cells. In general, transverse shear effects are not included in the analysis of lattices; however, shear effects can be included with additional mathematical terms. This and other lattice analysis drawbacks have prompted this study.

Objectives and Scope

This paper provides a fundamental and integral approach to the study of lattice truss structures. First, a review of current lattice analysis methods is presented. Next, lattice geometry, symmetry, topology, and design are examined. Then lattice behavior or mechanics are examined, and a new methodology for the analysis of lattice structures is presented. The methodology provides insight into lattice design for strength or stiffness. Additionally, emphasis is placed on exact solutions of various lattice parameters such as nodal displacements and member loads. This limits the scope of the study but allows for greater insight into the behavior of selected lattice geometries. The interested reader should see Ref. 9 for complete details, validation using the finite element code EAL, and additional lattice geometries.

Specific objectives of this research are 1) to develop truss beam geometries that under uniform loading exhibit classic fourth-, unique sixth-, and eighth-order behaviors for deflection and 2) to develop simple closed-form, exact deflection equations using Castigliano's second theorem over the nodal domain of the uniform lattice structures mentioned in objective 1.

Literature Review

The focus of much research in the past has been the analyses of lattice structures for civil application with various loading and boundary conditions for the acquisition of 1) member stress or strain values, 2) nodal deflection, or 3) lattice stiffness parameters. The leading lattice analysis methods are traditional methods from statics, energy methods, continuum modeling, and finite element analysis. Finite element analysis is generally used to solve specific lattice truss configurations and to provide detailed response determination. Developing discrete analysis models often requires significant effort for each configuration. The present approach examines selected configurations commonly used in lattice truss structure designs. The method provides design parameters related to stiffness and lattice behavior as well as expressions for deflection that are directly applicable to design optimization. These features are benefits not available from finite element analysis models. A description of each analysis method of a given type may be found in Refs. 10–15.

Lattice Designs

Traditional and modified versions of the Warren lattice design are the primary configurations examined in this study due to the extensive amount of previous work on similar lattice geometries. The typical planar Warren lattice geometry is shown in Fig. 1. Typical lattice beam geometries and terms are provided in Fig. 2. Bays are the repetitive lattice unit in two dimensions, and cells are the repetitive lattice unit in three dimensions. Lattice bays and cells consist of three types of members: longerons, diagonals, and battens. Longeron members usually lie parallel to the horizontal axis or plane. Battens usually lie parallel to the vertical axis, and diagonal members usually bisect the rectangle formed by the longeron and batten members. Each member is defined by its geometric location in the lattice. Surface members lie on the parallel upper and lower expanding horizontal planes of a lattice, and core members lie in between the two parallel surfaces.

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*Research Engineer, Thermal Structures Branch.
†Professor, Aerospace Engineering Department. Associate Fellow AIAA.

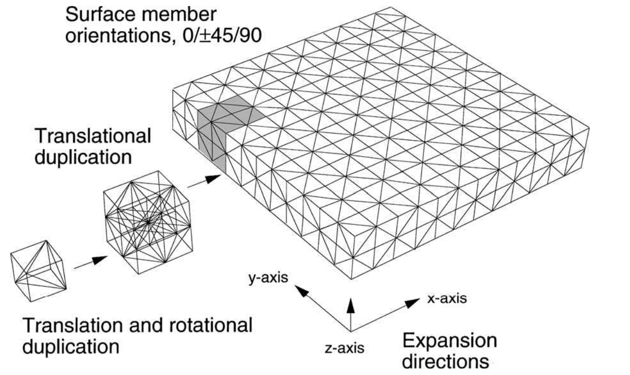
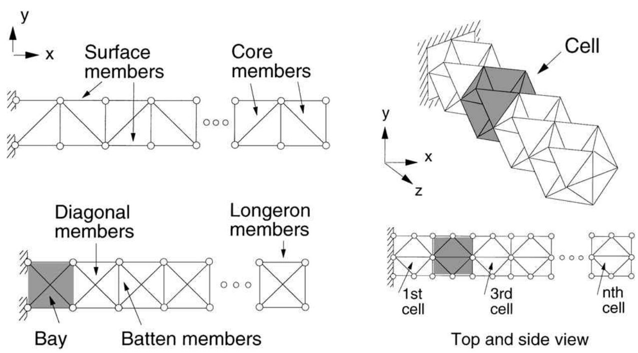


Fig. 1 Typical planar Warren lattice geometry.



Typical classic two-dimensional warren lattices

Typical soft three-dimensional lattice

Fig. 2 Typical two- and three-dimensional lattice beam geometry and members.

The types of lattice structures considered include 1) group 1, classic-beam-like lattices in both two and three dimensions, and 2) group 2, soft plate-like lattices in two and three dimensions. Beam-like lattices are readily defined; however, soft lattices are somewhat different.

Soft plate-like lattice geometries are derived from the basic Warren lattice geometry presented in Fig. 1 by the removal of critical surface longeron and batten members. These lattices are designed to be inherently weak in bending and to serve indirectly as test cases to assess the accuracy of the analysis method qualitatively.

Lattice Analysis Procedure

In two dimensions, constrained lattice structures, by definition, adhere to a minimum member m to node n relationship. This relationship for a two-dimensional lattice is $m = 2n$, and for a three-dimensional lattice the relationship is $m = 3n$. These equations are referred to as Maxwell's equations for structural stability or kinematic stability.[8] Lattices that have more than the required number of members have redundant members.

Static Determinacy

A statically determinate lattice structure exposed to external forces is defined as an internal system of pinned members and constraint arrangement from which internal forces and displacements can be determined exactly by the equations of statics, i.e., using the method of joints or method of sections.[8]

Castigliano's Second Theorem

To acquire statically determinate lattice behavior, a highly rigorous analysis method is desired. Knowing that the application of Castigliano's second theorem derived from a total complementary energy functional captures all of a material's displacement behavior due to point forces prompted its use. The generalized Castigliano's second theorem is presented mathematically as

$$\delta_i = \frac{\partial U'}{\partial P_i} \tag{1}$$

where U' is complementary strain energy, P is a generalized force, and δ is the generalized displacement in the direction of P. In words, Castigliano's second theorem states that the partial derivative of the complementary strain energy with respect to any independent generalized force P_i is equal to the generalized displacement δ_i located at the force P_i and in the direction of P_i. Equation (1) is simplified for this study due to the analysis of linearly elastic members at constant temperature. Hence, the complementary strain energy is equal to strain energy, $U = U'$, and Eq. (1) reduces to

$$\delta_i = \frac{\partial U}{\partial P_i} \tag{2}$$

This form of Castigliano's second theorm, utilized for this study, states that for a linear elastic system at constant temperature the partial derivative of the strain energy with respect to any independent generalized force P_i is equal to the generalized displacement δ_i located at the force P_i and in the direction of P_i.

Exact Deflection Derivation

To analyze statically determinate lattice groups 1 (classic beams) and 2 (soft, plate-like), a simple exact static analysis procedure, consisting of Castigliano's second theorem and extrapolation functions, is used. The analysis procedure consist of four main steps: 1) generating a lattice model with defined repeating bays or cells, boundary conditions, and specified loads; 2) calculating lattice member loads per bay or cell; 3) deriving nodal displacements per bay or cell using Castigliano's second theorem; and 4) deriving a displacement function using an extrapolation function for an exact fit.

To provide an understanding of the procedure, an example is presented. Figure 3a contains a cantilevered Pratt design lattice beam with an end load P and associated internal member loads. A Pratt lattice has two rows of longerons, vertical battens, and intermediate diagonal members. Member load results are obtained through any type of static analysis, i.e., the method of joints. Analysis of the

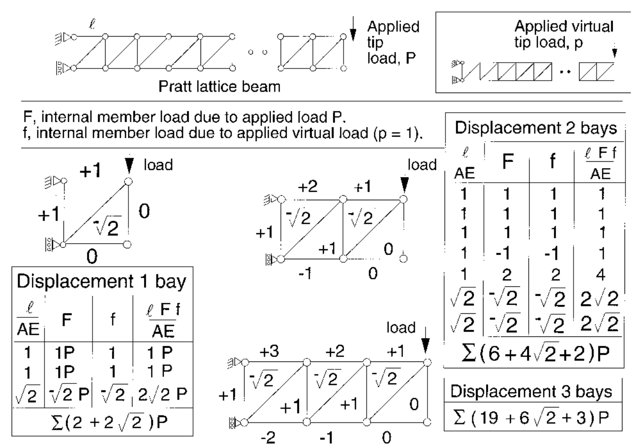


Fig. 3a Displacement equation derivation.

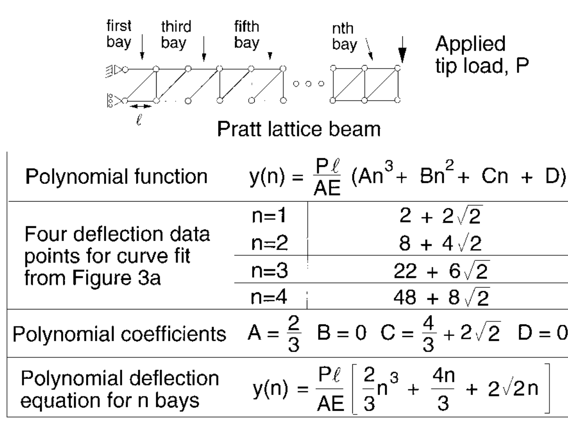


Fig. 3b Displacement equation derivation for a cantilevered Pratt lattice beam.

first three bays is illustrated on Fig. 3a. Analysis of the first bay is presented in the left-hand table. A deflection value δ is obtained by summing up the strain energy of the five members. This step is given by

$$\delta = \sum_{i=1} \frac{(\ell_i)(F_i)(f_i)}{A_i E_i} \tag{3}$$

where F_i is the member load due to actual forces; f_i is the member load due to a unit virtual load, applied at the desired point of displacement; and ℓ_i is the member length. This is an application of Castigliano's second theorem. The derived deflection value for one bay is presented at the bottom of the table in Fig. 3a. Analysis of a two-bay configuration is presented in the right-hand table. Again a deflection value is generated. Deflection values are generated through member load analysis of lattice beams with successively higher numbers of bays as indicated.

Generated deflection values for one, two, three, and four bays are presented in table form in Fig. 3b. As additional bays are analyzed, their contribution to the tip deflection is zero for more than four bays. Hence, the tip deflections for these four bays allow a fit to a cubic function in terms of the number of bays n. The derived exact deflection values are presented in the table in Fig. 3b for one through four bays. They are then fitted to a third-order polynomial function. Note that member load and deflection values are rational expressions due to geometry, boundary conditions, and the applied load. These are critical design and analysis criteria because the derivation of rational coefficients for the third-order extrapolation polynomial is possible only with rational displacement values. The resulting third-order deflection function, presented at the bottom of Fig. 3b, is given by

$$y(n) = \underbrace{\frac{P\ell}{AE}\left[\frac{2}{3}n^3\right]}_{\text{bending}} + \underbrace{\frac{P\ell}{AE}\left[\frac{4n}{3} + 2\sqrt{2}\,n\right]}_{\text{shear}} \tag{4}$$

which represents the exact vertical deflection equation of the node where the point load is applied for a finite number of bays n and can be verified through analytical methods or finite element programs (EAL, NASTRAN). Calculated polynomial coefficient terms, A–D given on Fig. 3b, are presented above the deflection equation. Note that the bending term, associated with the third-order term, results from surface longeron members, and the shear terms, associated with the first-order term, result from surface and core members. Polynomial coefficient terms, not shown in Eq. (4), represent deformation within the end bay of the truss, equivalent to St. Venant effects in a continuum. A third-order polynomial function is chosen as an extrapolation function because of the analogous solid mechanic solution for the end deflection of a tip-loaded cantilevered beam of length L given by

$$y = PL^3/3EI \tag{5}$$

and because surface member loads of the four-bay configuration increase from the applied load in a linear fashion. In general, the type of extrapolation or interpolation function is dependent on the change in member load from bay to bay or cell to cell. Because rational values can be fitted to any type of mathematical function, i.e., nth-order polynomial function or trigonometric function, this analysis has wide appeal for linear and nonlinear elastic material problems.

A comparison of the Pratt-derived displacement equation using strain energy and a solid mechanics beam displacement equation is presented to highlight the difference. The moment of inertia I of Eq. (5) is calculated using the parallel axis theorem:

$$I = 2Ad^2 = \tfrac{1}{2}A\ell^2 \tag{6}$$

where d is one-half the member length ℓ. Equation (6) substituted into Eq. (5) generates

$$y = 2P\ell/3AE \tag{7}$$

Therefore the classic beam derivation, Eq. (7), is the first term of lattice equation (3) with n equal to 1. An even more rigorous solution with shear terms can be produced using Timoshenko beam theory or a plane stress analysis with an Airy stress function.[9] However, in general, plane elasticity analysis is limited relative to strain energy analysis for lattice structures. The versatility of the presented energy method for other lattice structures is illustrated with additional example problems in subsequent sections. As a result of this analysis method, derived solutions are referred to as exact. The use of a finite element analysis computer program facilitates the derivation of member loads and validates derived equation results.

Analysis Results

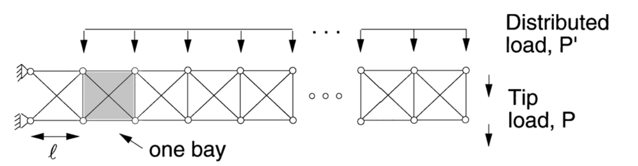
The cross-sectional area of a member is A, the term E represents Young's modulus of the member material, P represents an applied point load, and P' represents an applied distributed load. The unit length ℓ and the number of bays or cells n are defined relative to each lattice. The first group of lattices considered consists of cantilevered lattices in two and three dimensions that are analogous to classic beams and plates.

Quadrangular Warren Lattices in Two Dimensions

A two-dimensional quadrangular Warren lattice beam is presented in Fig. 4. It is derived from the Pratt lattice by including an additional diagonal member per bay. General deflection equations are presented for tip and distributed loadings. The diagonal member lacing pattern of the quadrangular lattice allows a more efficient load transfer than Pratt lacing; hence quadrangular shear terms are smaller. This lattice has one redundant member per bay, and lattice behavior is statically determinate only with the application of two equivalent point loads on vertically aligned nodes.

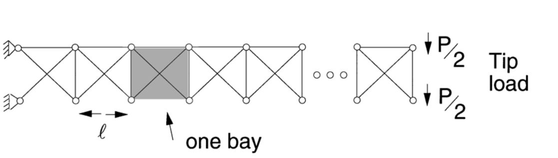
Modified Quadrangular Warren Lattice in Two Dimensions

A cantilevered modified quadrangular Warren lattice, where the bottom row of longerons has been removed, is presented in Fig. 5. This example is presented to illustrate the flexibility of lattice design and the capability of the analysis method for nontraditional lattice geometries. The highlighted gray area represents a typical repeating bay. A deflection equation for a tip load is presented in Fig. 5 below the lattice geometry. Note that longeron, batten, and diagonal core members contribute to the third-order bending terms and that the third-order coefficient for the square bay configuration is no longer two-thirds. Hence, the moment of inertia I of the lattice has changed. With this lattice geometry, the calculation of the moment of inertia term and subsequent Bernoulli–Euler beam deflection equations



Loading condition	Deflection equation
Tip load, P	$y(n) = \frac{P\ell}{AE}\left[\frac{2}{3}n^3 - \frac{n}{6} + \sqrt{2}\,n\right]$
Distributed load, P'	$y(n) = \frac{P'\ell}{AE}\left[\frac{n^4}{4} + \frac{n^3}{3} + \frac{n^2}{2}\sqrt{2}\right]$

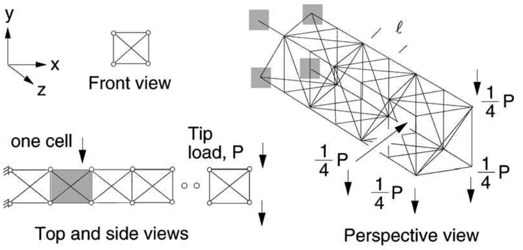
Fig. 4 Deflection equations of a quadrangular Warren lattice beam.



- unique member orientation
- moment of inertia term incalculable
- unique second order term

Deflection equation, tip load, P	$y(n) = \frac{P\ell}{EA}\left[\left[\frac{8}{3}n^3 - 2n^2 + \frac{1}{3}n\right] + \left[\left(\frac{4}{3}n^3 + \frac{2}{3}n\right)\sqrt{2}\right]\right]$

Fig. 5 Deflection equation of a modified two-dimensional quadrangular Warren lattice.



Deflection equation tip load, P	$y(n) = \frac{P\ell}{AE}\left[\frac{n^3}{3} + \frac{n}{6} + n\sqrt{2}\right]$

Fig. 6 Deflection equation of a three-dimensional Warren lattice beam.

may seem difficult. However, deflection equations are quite easily derived using strain energy methods.

Warren Lattice in Three Dimensions

The deflection equation and geometry of a three-dimensional Warren lattice beam with a tip load are presented in Fig. 6. One cell is highlighted in gray. The basic geometry of this lattice is derived by combining four two-dimensional Warren lattice beams with an interior diagonal member to form a rectangular cross section. The deflection equation is one-half of a two-dimensional Warren lattice beam deflection equation for comparable configurations due to loading conditions. This deflection equation is independent of beam width. Although the three-dimensional Warren lattice beam has one redundant member per bay, it behaves as two uncoupled two-dimensional Warren lattice beams due to the geometry and interaction of diagonal members. Note that redundant lattices require Warren diagonal member geometries and uniform loads for analysis. A redundant Pratt diagonal member geometry and off-center loads complicate three-dimensional deflection equation derivations because torsion and bending stiffnesses become coupled.

Soft Plate-Like Lattices

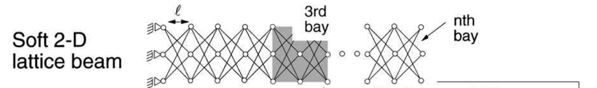
The second group of lattices considered consists of uniquely designed cantilevered soft lattices. Lattice behavior so far has been analogous to traditional elastic structures such as beams and plates. If Fig. 1 is revisited, and the removal of longeron and batten members is initiated, behavior unique to soft lattice structures occurs.

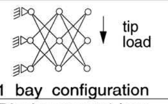
Sixth-Order Lattice in Two Dimensions

With the removal of all longeron and batten members from a double-layer quadrangular lattice and the addition of diagonal members, a two-dimensional soft lattice is generated. As in Figs. 3a and 3b, to derive the governing deflection equation, selected bays are analyzed for displacement values. Figure 7a contains the derivation and displacement values for the first two bays. Four additional values are presented in Fig. 7b. After analysis, a fifth-order term arises in the deflection equation with a tip load, and a sixth-order term arises in the deflection equation with a distributed load, as given at the bottom of Fig. 7b, due to the altered stiffness of the lattice design. This is unique in that traditional tip-loaded cantilevered beam deflection equations are of the third order. Note that lower-order terms have been omitted from both deflection equations. However, examination of the lattice geometry, in contrast to the beams presented in Figs. 3–5, illustrates a very weak geometry for load transfer in bending. In general, cantilevered elastic beams deflect downward and rotate. To contrast, a cantilevered soft sixth-order lattice geometry induces side moments along the uppermost and lowermost nodes that inwardly warp the lattice and create an exaggerated Poisson's ratio effect. Hence, as the lattice deflects downward, upper and lower nodes remain plane and rotate, and the distance between upper and lower surface nodes decreases. This additional effect leads to a fifth-order deflection term. To parallel fourth-order beam theory in solid mechanics, lattices with sixth-order deflection equations due to distributed loading are referred to as soft sixth-order lattices.

Sixth-Order Lattice in Three Dimensions

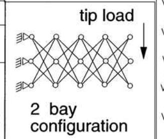
The next soft lattice is three dimensional. The lattice is examined with a distributed and an end load. The lattice is presented in Fig. 8.





Displacement 1 bay

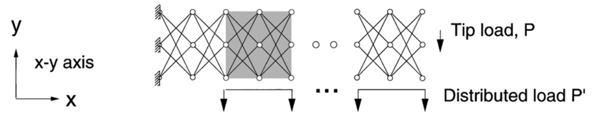
$\frac{\ell}{AE}$	F	f	$\frac{\ell F f}{AE}$
$\sqrt{2}$	$\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\sqrt{2}$	$\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\sqrt{2}$	$3\sqrt{2}/2$	$3\sqrt{2}/2$	$9\sqrt{2}/2$
$\sqrt{2}$	$3\sqrt{2}/2$	$3\sqrt{2}/2$	$9\sqrt{2}/2$
$\sqrt{5}$	$\sqrt{5}$	$\sqrt{5}$	$5\sqrt{5}$
$\sqrt{5}$	$\sqrt{5}$	$\sqrt{5}$	$5\sqrt{5}$
$\Sigma(10\sqrt{2} + 10\sqrt{5})$			



Displacement 2 bays

$\frac{\ell}{AE}$	F	f	$\frac{\ell F f}{AE}$	2x members
$\sqrt{2}$	$\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}$
$\sqrt{2}$	$\frac{3\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{2}$	$9\sqrt{2}/2$	$9\sqrt{2}$
$\sqrt{2}$	$4\sqrt{2}$	$4\sqrt{2}$	$32\sqrt{2}$	$64\sqrt{2}$
$\sqrt{2}$	$\frac{3\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{2}$	$9\sqrt{2}/2$	$9\sqrt{2}$
$\sqrt{2}$	$4\sqrt{2}$	$4\sqrt{2}$	$32\sqrt{2}$	$64\sqrt{2}$
$\sqrt{2}$	$\frac{15\sqrt{2}}{2}$	$\frac{15\sqrt{2}}{2}$	$\frac{225\sqrt{2}}{2}$	$225\sqrt{2}$
$\sqrt{5}$	$\sqrt{5}$	$\sqrt{5}$	$5\sqrt{5}$	$10\sqrt{5}$
$\sqrt{5}$	$3\sqrt{5}$	$3\sqrt{5}$	$45\sqrt{5}$	$90\sqrt{5}$
$\sqrt{5}$	$6\sqrt{5}$	$6\sqrt{5}$	$180\sqrt{5}$	$360\sqrt{5}$
$\Sigma(372\sqrt{2} + 460\sqrt{5})$				

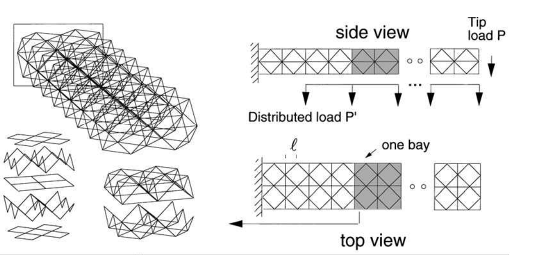
Fig. 7a Soft lattice deflection equation derivation.



Polynomial function	$y(n) = \frac{P\ell}{AE}(An^5 + Bn^4 + Cn^3 + Dn^2 + En + F)$			
Six data points for curve fit, Figure 7a	n=3	$2974\sqrt{2} + 3710\sqrt{5}$	n=5	$39346\sqrt{2} + 49170\sqrt{5}$
	n=4	$12776\sqrt{2} + 15960\sqrt{5}$	n=6	$98396\sqrt{2} + 122980\sqrt{5}$
Polynomial coefficients	$\sqrt{2}$	A = 16 B = 0 C = $-20/3$ D = 0 E = $2/3$ F = 0		
	$\sqrt{5}$	A = $64/5$ B = 0 C = $-16/3$ D = 0 E = $38/15$ F = 0		
Deflection equation tip load, P	$y(n) = \frac{P\ell}{AE}\left[(192n^5 - 80n^3)\frac{\sqrt{2}}{15} + (48n^5 - 20n^3)\frac{\sqrt{5}}{3}\right]$			

Deflection equation distributed load, P'	$y(n) = \frac{P'\ell}{AE}\left[(160n^6 + 288n^5)\frac{\sqrt{2}}{45} + (80n^6 + 144n^5)\frac{\sqrt{5}}{18}\right]$

Fig. 7b Displacement equation derivation for a cantilevered two-dimensional soft lattice beam.



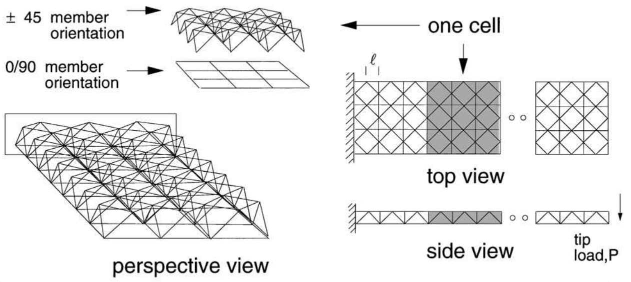
Deflection equation tip load, P	$y(n) = \frac{P\ell}{AE}\left[15360n^5 + 2432n^3 - 960n^2\right]\frac{\sqrt{2}}{1920}$
Deflection equation distributed load, P'	$y(n) = \frac{P'\ell}{AE}\left[1280n^6 + 1664n^4 - 576n^3\right]\frac{\sqrt{2}}{576}$

Fig. 8 Deflection equations of a three-dimensional soft sixth-order Warren lattice.

The lattice geometric cell is four bays wide and two bays deep. One cell is isolated, and selected parts are shown. Another cell is highlighted in gray on the side view. The repeating cell consists of four cubes of diagonal surface members with a midplane of longeron members. Examination of the lattice cross section, in contrast to the three-dimensional Warren beam presented in Fig. 6, illustrates a very weak geometry for load transfer in bending. To contrast, this lattice geometry induces side moments along the lengthwise edges that inwardly warp the lattice and create an exaggerated Poisson's ratio effect. Hence, nodes deflect downward, widthwise cross sections remain plane and rotate, and lengthwise edge nodes rotate inward. This additional rotation leads to the addition of a fifth-order deflection term.

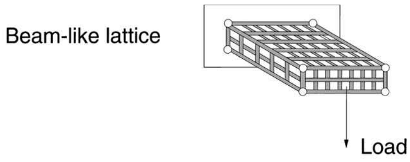
Eighth-Order Lattice in Three Dimensions

An asymmetric soft lattice is derived by removing the bottom half of the previous soft lattice and expanding by two bays. Lattice cells are six bays wide and one bay high. The repeating cell of this lattice



Deflection equation, tip load, P	$y(n) = \frac{P\ell}{AE}\left[\left[\frac{4374}{5}n^5 + \frac{34560}{5}n^3\right] + \left[43740n^7 + 42525n^5 + 75880n^3\right]\frac{\sqrt{2}}{5040}\right]$

Fig. 9 Deflection equation for a three-dimensional soft eighth-order Warren lattice with end load.



		Displacement Field			
		2-D (x-y)		3-D (x-y-z)	
Load		tip	distributed	tip	distributed
Lattice type	Classic	3rd	4th	3rd	4th
	Soft	5th	6th	5th and 7th	6th and 8th

Fig. 10 Summary of derived polynomial deflection equations.

and the associated deflection equation for a tip load are presented in Fig. 9. With a tip load uniformly distributed over the lattice width, this lattice exhibits seventh-order deflection behavior due to the geometry of the top surface diagonal members. This behavior is a result of the top ±45-deg and bottom 0,90-deg member orientations that induce side moments and widthwise axial compression. Therefore, a seventh-order deflection equation results. An eighth-order equation is presumed to occur for a distributed loading case. Again, to parallel fourth-order beam theory in solid mechanics, lattices with seventh-order deflection equations due to a tip load are referred to as eighth-order lattices.

Classic and Soft Lattice Mechanics

To gain more insight into soft lattice mechanics, related classic governing geometric deformation relations are discussed. Governing two-dimensional strain-displacement relations or kinematic relations[12] for linear elasic material are valid for soft lattice structures. However, strain-curvature relations are invalid for soft lattice structures. This leads one to postulate that soft lattices uniquely represent the material zone between membranes that have in-plane stiffness and no out-of-plane stiffness and plates that have in-plane stiffness and out-of-plane, fourth-order bending stiffness. Hence, a difference between classic elastic materials and soft lattices is identified. However, note that the governing equations of soft lattices (i.e., governing differential equations, boundary conditions, and strain-curvature relations) and limits of higher-order soft lattice theory have yet to be derived. The derived classic and soft lattice deflection behaviors are compared and summarized in Fig. 10.

Conclusions

The use of strain energies to achieve the two objectives has been demonstrated by numerous examples. On the foundation of these examples the following conclusions are drawn. First, closed-form deflection equations for statically determinate lattice beams consist of exact third- and fourth-order polynomial functions in bending and lower-order polynomial terms in shear for static analysis. Second, lattice geometries that exhibit sixth- and eighth-order deflection equations are generated by expanding the width of a lattice beam and removing redundant members. Deflection equations are derived by increasing the order of the polynomial interpolation function.

The derived deflection equations also provide an extended view of elastic material fundamentals. Classical strain-curvature relationships and deflection equations are generated for beams, plates, or shells under uniform loading with small or large deflection assumptions, governing partial differential equations and boundary conditions; note that finite element analysis cannot generate closed-form solutions. The presented analysis does not have all of these distinct assumptions, and hence a wider range of solutions can be derived as shown. However, with these solutions two critical questions arise: Can a differential equation and boundary conditions associated with a higher-order deflection equation be generated, and does an upper limit exist for the polynomial order of a deflection equation in a specified two-dimensional or three-dimensional field? Hence, this study of elastic lattices with different geometries provides a unique view of linear elastic material behavior.

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R. K. Kapania
Associate Editor